Nonthermal and Plasmon Effects on Elastic Electron-Ion Collisions in Hot Quantum Lorentzian Plasmas

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The nonthermal and plasmon effects on elastic electron-ion collisions are investigated in hot quantum Lorentzian plasmas. The modified interaction model taking into account the nonthermal screening and plasmon effects is employed to represent the electron-ion interaction potential in hot quantum Lorentzian plasmas. The eikonal phase and differential collision cross-section are obtained as functions of the impact parameter, collision energy, spectral index, and plasma parameters by using the second-order eikonal analysis. It is shown that the plasmon effect suppresses the eikonal phase and collision cross-section for $0 < \beta$ $(\equiv \hbar\omega_0/k_B T < 0.6)$ and, however, enhances it for $0.6 < \beta < 1$, where ω_0 is the plasma frequency and T is the plasma temperature. It is also shown that the nonthermal character of the quantum Lorentzian plasma suppresses the elastic electron-ion collision cross-section.

Key words: Nonthermal Effects; Plasmon Effects; Quantum Lorentzian Plasmas.

The elastic electron-ion collision [1] has been receiving much attention since this is one of the major atomic processes and also has applications in many areas of physics. Recently, atomic collision and radiation processes [2 – 5] have been widely used as plasma diagnostics in various plasmas such as weakly and strongly coupled plasmas. It is known that the characteristic features of plasmas would be comprehended by exploring the velocity distribution of plasma particles. The classical Boltzmann plasma is in thermal equilibrium which implies that there would be no energy exchanges between the charged particles in plasmas. However, coupling of the external field with the equilibrium plasma most often generates superthermal electrons departed from the ordinary Boltzmann velocity distribution in various astrophysical and laboratory plasmas [6 – 8]. In addition, the multiparticle correlation effects caused by concurrent interactions of many plasma particles due to an increase of the plasma density should be taken into account to characterize the interaction potential. In these circumstances, the interaction potential would not be represented by the classical Debye-Hückel potential obtained by the classical Boltzmann velocity distribution of charged particles because of the plasmon effects caused by the collective plasma oscillations in hot quantum plasmas [9]. In recent years, there has been a considerable interest in quantum effects in plasmas [8 – 10] since quantum plasmas have been shown in dense astrophysical plasmas, in various nano devices such as quantum dot, and in laser-produced dense laboratory plasmas [10]. Thus, it would be expected that atomic collisions in hot quantum nonthermal plasmas would be unquestionably different from those in classical Debye-Hückel plasmas.

Thus, in the present paper we investigate the nonthermal and plasmon effects on the elastic electron-ion collision in hot quantum Lorentzian plasmas. The modified interaction model [7] taking into account the nonthermal screening and plasmon effects is engaged to represent the electron-ion interaction in hot quantum Lorentzian plasmas. The second-order eikonal method with the impact parameter analysis is applied to obtain the eikonal phase and collision cross-section as functions of the impact parameter, collision energy, plasma parameters, and spectral index of the plasma.

The solution $\psi_k(\mathbf{r})$ of the Schrödinger equation for the interaction potential $V(\mathbf{r})$ can be represented by the following integral form of the Lippmann-Schwinger equation [1]:

$$\psi_k(\mathbf{r}) = \varphi_k(\mathbf{r}) + \frac{2\mu}{\hbar^2} \int \mathrm{d}^3\mathbf{r}' G(\mathbf{r},\mathbf{r}') V(\mathbf{r}') \psi_k(\mathbf{r}'), \quad (1)$$

where $\varphi_k(\mathbf{r})$ and $G(\mathbf{r},\mathbf{r}')$ are the solution of the homogeneous equation and Green's function, respectively,

$$(\nabla^2+k^2)\varphi_k(\mathbf{r})=0, \quad (2)$$

$$(\nabla^2+k^2)G(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r},\mathbf{r}'), \quad (3)$$

k [$=(2\mu E/\hbar^2)^{1/2}$] is the wave number, μ the reduced mass of the collision system, E ($=\mu v^2/2$) the collision energy, v the relative collision velocity, and $\delta(\mathbf{r},\mathbf{r}')$ the delta-function. The solution of (2) is then given by

$$\psi_k^{(+)}(\mathbf{r})\cong\frac{\mathrm{e}^{\mathrm{i}k\cdot\mathbf{r}}}{(2\pi)^{3/2}} \cdot\left[1+\frac{2\mu}{\hbar^2}\int \mathrm{d}^3\mathbf{r}'G^{(+)}(\mathbf{r},\mathbf{r}')V(\mathbf{r}')\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\right], \quad (4)$$

where $G^{(+)}(\mathbf{r},\mathbf{r}')$ ($=-\mathrm{e}^{-\mathrm{i}|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$) is the free outgoing Green's function [11]. It is shown that the validity condition of the eikonal method is known as $|V|/E<1$ [11], where $|V|$ is a typical strength of the interaction potential. Introducing cylindrical coordinates such as $\mathbf{r}=\mathbf{b}+z\hat{\mathbf{n}}$, where $\mathbf{b}$ is the impact parameter, $\hat{\mathbf{n}}$ is the unit vector transverse to the momentum transfer $\mathbf{K}$ ($\equiv\mathbf{k}_i-\mathbf{k}_f$), $\mathbf{k}_i$ and $\mathbf{k}_f$ are the incident and final wave vectors, respectively, the eikonal scattering amplitude $f_{\mathrm{E}}(\mathbf{K})$ is then represented by

$$f_{\mathrm{E}}(\mathbf{K})=-\frac{\mu}{2\pi\hbar^2}\int \mathrm{d}^3\mathbf{r}\mathrm{e}^{\mathrm{i}\mathbf{K}\cdot\mathbf{r}}V(\mathbf{r}) \cdot\exp\left[-\frac{\mathrm{i}\mu}{\hbar^2 k_i}\int_{-\infty}^{z}\mathrm{d}z'V(\mathbf{b},z')\right]. \quad (5)$$

Since the differential eikonal collision cross-section is determined by the relation $\mathrm{d}\sigma_{\mathrm{E}}/\mathrm{d}\Omega=|f_{\mathrm{E}}(\mathbf{K})|^2$, the total elastic eikonal collision cross-section σ_{E} can be written as

$$\sigma_{\mathrm{E}}(k)=\int \mathrm{d}^2\mathbf{b}|\exp[\mathrm{i}\chi_{\mathrm{E}}(b,k)]-1|^2, \quad (6)$$

where $\mathrm{d}\Omega$ is the differential solid angle and the eikonal phase $\chi_{\mathrm{E}}(b,k)$ can be expressed as the following series expansion form [11, 12]:

$$\chi_{\mathrm{E}}(b,k)=\sum_l\frac{1}{(l+1)!}\left(-\frac{\mu}{\hbar^2}\right)^{l+1} \cdot\frac{1}{k}\left(\frac{\partial}{\partial k}\frac{1}{k}-\frac{b}{k^2}\frac{\partial}{\partial b}\right)^l\int_{-\infty}^{\infty}\mathrm{d}zV^{l+1}(b,z) \quad (7)$$

with $|\mathbf{k}_i|=|\mathbf{k}_f|=k$.

In many astrophysical and laboratory plasmas, the important departures from the equilibrium Boltzmann velocity distribution would be anticipated due to the strong external disturbances. These so-called superthermal electrons escape the ordinary Boltzmann distribution corresponding to the bulk of plasma electrons, which can be modeled more effectively by the Lorentzian velocity distribution [6, 8]. Moreover, an excellent work by Hasegawa et al. [6] has certified that the equilibrium plasma distribution function in the presence of a superthermal radiation field resembles the Lorentzian distribution function. In these Lorentzian plasmas [8], the characteristic energy E_κ is represented by $E_\kappa=[(\kappa-3/2)/\kappa]E_T$, where κ ($>3/2$) is the spectral index of the plasma, $E_T\equiv k_{\mathrm{B}}T$, k_{B} is the Boltzmann constant, and T is the plasma temperature. It is also shown that the Debye radius L_κ in Lorentzian plasmas [8] including the nonthermal character is given by $L_\kappa=[(\kappa-3/2)/(\kappa-1/2)]^{1/2}L$, where L is the ordinary Debye radius for the Boltzmann distribution. In addition, the remarkably useful analytical form of the modified interaction potential [9, 13] in hot quantum plasmas has been obtained by the quantum approach including the plasmon effects caused by strong plasma oscillations. These quantum effects may complicate the picture of the screened Yukawa-type Debye-Hückel interaction between charged particles in plasmas. Using this modified interaction model [7] with the plasma parameters for the Lorentzian distribution, the interaction potential V_{QL} between an electron and ion with charge Ze in a hot quantum Lorentzian (QL) plasma can be obtained by

$$V_{\mathrm{QL}}(r,\kappa,\beta,L)=-\frac{Ze^2}{r}\frac{1}{4(1-\beta_\kappa^2)^{1/2}}\Big\{(4-\beta_\kappa)\mathrm{e}^{-r/L_{1,\kappa}} -2[1-(1-\beta_\kappa^2)^{1/2}]\mathrm{e}^{-r/L_{2,\kappa}}\Big\}, \quad (8)$$

where the characteristic parameter β_κ is given by $\beta_\kappa\equiv[\kappa/(\kappa-3/2)]\beta$, $\beta\equiv\hbar\omega_0/E_T$, $\hbar\omega_0$ is the plasmon energy, ω_0 is the plasma frequency, $L_{1,\kappa}\equiv[1+(1-\beta_\kappa^2)^{1/2}]^{1/2}(L_\kappa/2^{1/2})$, $L_{2,\kappa}\equiv[1-(1-\beta_\kappa^2)^{1/2}]^{1/2}(L_\kappa/2^{1/2})$, and $r=(b^2+z^2)^{1/2}$. This potential would be valid for $0\le\beta_\kappa<\kappa/(\kappa-3/2)$ since the plasmon energy $\hbar\omega_0$ is expected to be smaller than E_T in the modified interaction model [7]. If nonthermal and plasmon effects are absent, the effective interaction potential (8) goes over into the case of the classical Debye-Hückel (DH) potential $V_{\mathrm{QL}}\to V_{\mathrm{DH}}=(-Ze^2/r)\mathrm{e}^{-r/L}$ since $L_{1,\kappa}\to L$ and $L_{2,\kappa}\to 0$ as

$\kappa \to \infty$ and $\beta \to 0$. The comprehensive and detailed discussion on the mechanisms of plasmon-particle and plasmon-plasmon collisions and the decay process of the plasmon can be found in an excellent work by Tsytovich [14]. After some mathematical manipulations by using (7) and (8) with the identity of the nth-order of the MacDonald function K_n [15], $K_n(z) = [\pi^{1/2}/(n-1/2)!](z/2)^n \int_1^\infty \mathrm{d}t \mathrm{e}^{-zt}(t^2-1)^{n-1/2}$, the total eikonal phase χ_E retaining the first- and second-order contributions for the elastic election-ion collision in hot quantum Lorentzian plasmas is found to be

$$\begin{aligned}\chi_\mathrm{E}(\bar{b},\bar{E},\kappa,\beta) = & \frac{\bar{E}^{-1/2}}{2(1-\beta_\kappa^2)^{1/2}}\Big\{(4-\beta_\kappa)K_0(\bar{L}_{1,\kappa}^{-1}\bar{b}) \\ & -2[1-(1-\beta_\kappa^2)^{1/2}]K_0(\bar{L}_{2,\kappa}^{-1}\bar{b})\Big\} \\ & +\frac{\bar{E}^{-3/2}}{8(1-\beta_\kappa^2)^{1/2}}\Big\{(4-\beta_\kappa)^2\bar{L}_{1,\kappa}^{-1}K_0(2\bar{L}_{1,\kappa}^{-1}\bar{b}) \\ & -2(4-\beta_\kappa)[1-(1-\beta_\kappa^2)^{1/2}](\bar{L}_{1,\kappa}^{-1}+\bar{L}_{2,\kappa}^{-1})K_0[(\bar{L}_{1,\kappa}^{-1} \\ & +\bar{L}_{2,\kappa}^{-1})\bar{b}]+4[1-(1-\beta_\kappa^2)^{1/2}]^2\bar{L}_{2,\kappa}^{-1}K_0(2\bar{L}_{2,\kappa}^{-1}\bar{b})\Big\},\end{aligned} \tag{9}$$

where $\bar{b}$ $(\equiv b/a_Z)$ is the scaled impact parameter, a_Z $(= a_0/Z)$ the Bohr radius of the hydrogen ion with nuclear charge Ze, a_0 $(= \hbar^2/me^2)$ the Bohr radius of the hydrogen atom, m the electron mass, $\bar{E}$ $(\equiv E/Z^2Ry)$ the scaled collision energy, and Ry $(= me^4/2\hbar^2 \approx 13.6\,\mathrm{eV})$ the Rydberg constant. The scaled screening radii $\bar{L}_{1,\kappa}$ and $\bar{L}_{2,\kappa}$ are $\bar{L}_{1,\kappa}$ $(\equiv L_{1,\kappa}/a_Z) = [(\kappa-3/2)/(\kappa-1/2)]^{1/2}[1+(1-(\kappa/(\kappa-3/2))^2\beta^2)^{1/2}]^{1/2}(\bar{L}/2^{1/2})$ and $\bar{L}_{2,\kappa}$ $(\equiv L_{2,\kappa}/a_Z) = [(\kappa-3/2)/(\kappa-1/2)]^{1/2}[1-(1-(\kappa/(\kappa-3/2))^2\beta^2)^{1/2}]^{1/2}(\bar{L}/2^{1/2})$, and $\bar{L}$ $(\equiv L/a_Z)$ is the scaled ordinary Debye radius. Thus, the scaled differential collision cross-section $\partial\bar{\sigma}_\mathrm{E}$ $[\equiv (\mathrm{d}\sigma_\mathrm{E}/\mathrm{d}\bar{b})/\pi a_0^2]$ in units of πa_0^2 within the framework of the second-order eikonal analysis for the elastic electron-ion collision in hot quantum Lorentzian plasmas is obtained as

$$\begin{aligned}\partial\bar{\sigma}_\mathrm{E} = & 2\bar{b}\big|\exp[\mathrm{i}\chi_\mathrm{E}(\bar{b},\bar{E},\kappa,\beta)]-1\big|^2 = \\ & 2\bar{b}\Big|\exp\Big\{(\mathrm{i}/2)(1-\beta_\kappa^2(\kappa,\beta))^{-1/2}\bar{E}^{-1/2}\Big[(4 \\ & -\beta_\kappa(\kappa,\beta))K_0(\bar{L}_{1,\kappa}^{-1}(\kappa,\bar{L})\bar{b})-2\big[1-(1 \\ & -\beta_\kappa^2(\kappa,\beta))^{1/2}\big]K_0(\bar{L}_{2,\kappa}^{-1}(\kappa,\bar{L})\bar{b})\Big]+(\mathrm{i}/8)(1 \\ & -\beta_\kappa^2(\kappa,\beta))^{-1/2}\bar{E}^{-3/2}\Big[(4 \\ & -\beta_\kappa(\kappa,\beta))^2\bar{L}_{1,\kappa}^{-1}(\kappa,\bar{L})K_0(2\bar{L}_{1,\kappa}^{-1}(\kappa,\bar{L})\bar{b}) \\ & -2(4-\beta_\kappa(\kappa,\beta))\big[1-(1-\beta_\kappa^2(\kappa,\beta))^{1/2}\big](\bar{L}_{1,\kappa}^{-1}(\kappa,\bar{L}) \\ & +\bar{L}_{2,\kappa}^{-1}(\kappa,\bar{L}))K_0((\bar{L}_{1,\kappa}^{-1}(\kappa,\bar{L})+\bar{L}_{2,\kappa}^{-1}(\kappa,\bar{L}))\bar{b}) \\ & +4\big[1-(1-\beta_\kappa^2(\kappa,\beta))^{1/2}\big]^2\bar{L}_{2,\kappa}^{-1}(\kappa,\bar{L}) \\ & \cdot K_0(2\bar{L}_{2,\kappa}^{-1}(\kappa,\bar{L})\bar{b})\Big]\Big\}-1\Big|^2.\end{aligned} \tag{10}$$

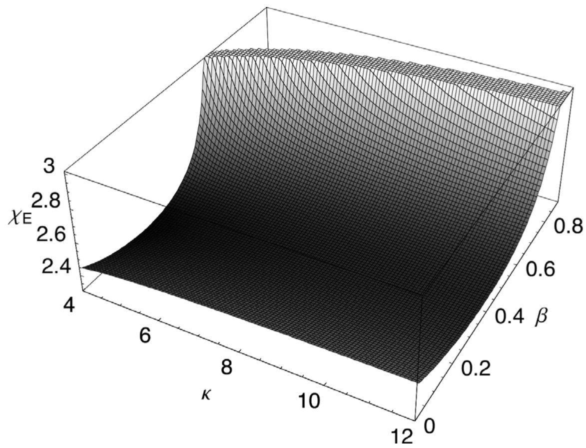

Fig. 1. The three-dimensional plot of the total eikonal phase χ_E as a function of the spectral index κ and plasmon parameter β for $\bar{b} = 2$, $\bar{E} = 10$, and $\bar{L} = 100$.

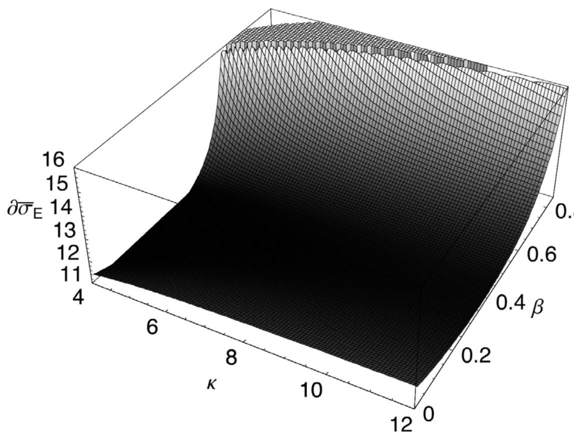

Fig. 2. The three-dimensional plot of the scaled differential collision cross-section $\partial\bar{\sigma}_\mathrm{E}$ as a function of the spectral index κ and plasmon parameter β for $\bar{b} = 2$, $\bar{E} = 10$, and $\bar{L} = 50$.

It has been shown that the quantum tunneling effects are important when the de Broglie length of the particle is comparable to the dimension of the system in quantum plasmas [16]. Recently, an excellent investigation on the formation and dynamics of solitons and vortices in quantum plasmas was given by Shukla and Eliasson [16]. In addition, an excellent discussion on the potential of a moving test charge in quantum plasmas including

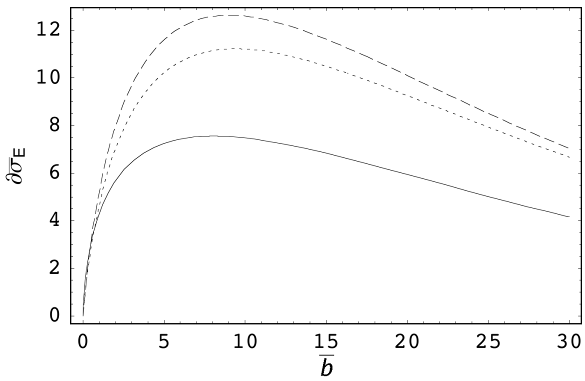

Fig. 3. The scaled differential collision cross-section $\partial\bar{\sigma}_E$ as a function of the impact parameter $\bar{b}$ for $\beta = 0.6$, $\bar{E} = 20$, and $\bar{L} = 50$. The solid line represents the case of $\kappa = 3$. The dashed line represents the case of $\kappa = 5$. The dotted line represents the case of $\kappa = 10$.

the quantum Bohm effect was given by Ali and Shukla [17].

In order to explicitly investigate the nonthermal and plasmon effects on the elastic electron-ion collision in hot quantum Lorentzian plasmas, we consider $\bar{E} > 1$ since the eikonal method is known to be valid for high-collision velocities [11]. From (10), it is explicitly shown that the eikonal cross-section depends on the details of the plasmon and nonthermal effects of hot quantum Lorentzian plasmas. Figure 1 presents the three-dimensional plot of the total eikonal phase χ_E as a function of the spectral index κ and plasmon parameter β. From this figure, it is shown that the plasmon effect suppresses the eikonal phase for $0 < \beta\ (\equiv \hbar\omega_0/k_B T) < 0.6$ and, however, enhances it for $0.6 < \beta < 1$. Figure 2 shows the three-dimensional plot of the scaled differential collision cross-section $\partial\bar{\sigma}_E$ as a function of the spectral index κ and plasmon parameter β. As it is seen, the plasmon effect also suppresses the differential collision cross-section for $0 < \beta < 0.6$ and enhances it for $0.6 < \beta < 1$. Thus, it is interesting to note that the cross-section increases until the plasmon energy $\hbar\omega_0$ is roughly equal to the characteristic energy E_κ of the Lorentzian plasma. Figure 3 presents the scaled differential collision cross-section $\partial\bar{\sigma}_E$ as a function of the scaled impact parameter $\bar{b}$ for various values of the spectral index κ. As we see in this figure, the eikonal collision cross-section increases with increasing the spectral index. Thus, it should be noted that the nonthermal character of the quantum Lorentzian plasma suppresses the elastic electron-ion collision cross-section. In addition, it is found that the nonthermal effects are more significant near the maximum positions of the differential collision cross-sections and decrease with an increase of the impact parameter.

From these results, we have found that the nonthermal and plasmon effects play a very important role in the elastic electron-ion collision in hot quantum Lorentzian plasmas. These results would provide useful information concerning the nonthermal and plasmon effects on the atomic collision processes in hot quantum nonthermal plasmas.

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